

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

A-level FURTHER MATHEMATICS

Paper 1

Wednesday 22 May 2024

Afternoon

Time allowed: 2 hours

Materials

- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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17	
18	
TOTAL	



Answer **all** questions in the spaces provided.

- 1** The roots of the equation $20x^3 - 16x^2 - 4x + 7 = 0$ are α, β and γ

Find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Circle your answer.

[1 mark]

$$-\frac{4}{5}$$

$$-\frac{1}{5}$$

$$\frac{1}{5}$$

$$\frac{4}{5}$$

- 2** The complex number $z = e^{\frac{i\pi}{3}}$
- Which **one** of the following is a real number?

Circle your answer.

[1 mark]

$$z^4$$

$$z^5$$

$$z^6$$

$$z^7$$



- 3 The function f is defined by

$$f(x) = x^2 \quad (x \in \mathbb{R})$$

Find the mean value of $f(x)$ between $x = 0$ and $x = 2$

Circle your answer.

[1 mark]

$$\frac{2}{3}$$

$$\frac{4}{3}$$

$$\frac{8}{3}$$

$$\frac{16}{3}$$

- 4 Which **one** of the following statements is correct?

Tick (✓) **one** box.

[1 mark]

$$\lim_{x \rightarrow 0} (x^2 \ln x) = 0$$

☐

$$\lim_{x \rightarrow 0} (x^2 \ln x) = 1$$

☐

$$\lim_{x \rightarrow 0} (x^2 \ln x) = 2$$

☐

$$\lim_{x \rightarrow 0} (x^2 \ln x) \text{ is not defined.}$$

☐

Turn over for the next question

Turn over ►



[3 marks]

[illegible]

[2 marks]

[illegible]

6

$$u_{n+1} = u_n + 3n$$

Prove by induction that for all integers $n \geq 1$

[4 marks]

[illegible]

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Turn over ►



- 7 The complex numbers z and w satisfy the simultaneous equations

$$z + w^* = 5$$

$$3z^* - w = 6 + 4i$$

Find z and w

[5 marks]



[4 marks]

The line with equation $y = mx + 4$ is a tangent to E

Without using differentiation show that $m = \pm\sqrt{7}$

[illegible]

$$p = \ln(r + \sqrt{r^2 + 1})$$

[4 marks]

[illegible]

[4 marks]

[illegible]

[6 marks]

[illegible]

[1 mark]

[4 marks]

[illegible]

The line L_1 has equation

$$\mathbf{r} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 6 \\ -5 & 2 & -3 \end{bmatrix}$$

The transformation T transforms the line L_1 to the line L_2

Show that the angle between L_1 and L_2 is 0.701 radians, correct to three decimal places.

[4 marks]

[illegible]

[6 marks]

[illegible]

13 (a) Use de Moivre's theorem to show that

$$\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$$

[3 marks]

13 (b) Use de Moivre's theorem to express $\sin 3\theta$ in terms of $\sin \theta$

[2 marks]



$$\cot 3\theta = \frac{\cot^3 \theta - 3\cot \theta}{3\cot^2 \theta - 1}$$
[illegible]

$$\frac{dy}{dx} + y \tanh x = \sinh^3 x$$

[7 marks]

[illegible]

[illegible]

Turn over ►



A curve is defined parametrically by the equations

$$y = t^{\frac{9}{2}} \quad (t \geq 0)$$

Show that the arc length of the curve from $t = 0$ to $t = 2$ is equal to 26 units.

[illegible]

[illegible]

Turn over ►



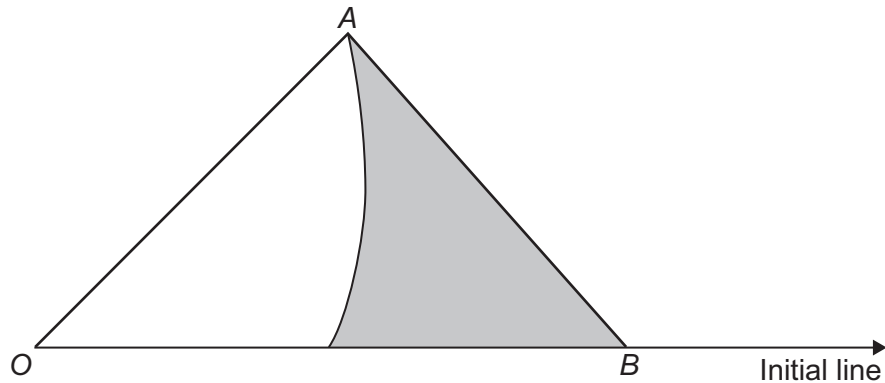
16

The curve C has polar equation $r = 2 + \tan \theta$

The curve C meets the line $\theta = \frac{\pi}{4}$ at the point A

The point B has polar coordinates $(4, 0)$

The diagram shows part of the curve C , and the points A and B

**16 (a)**

Show that the area of triangle OAB is $3\sqrt{2}$ units.

[2 marks]

16 (b)

Find the area of the shaded region.

Give your answer in an exact form.

[7 marks]



[illegible]

Turn over ►



By making a suitable substitution, show that

[7 marks]

[illegible]

[illegible]

Turn over ►

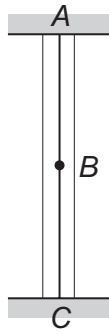


18 In this question use $g = 9.8 \text{ m s}^{-2}$

Two light elastic strings each have one end attached to a small ball B of mass 0.5 kg

The other ends of the strings are attached to the fixed points A and C , which are 8 metres apart with A vertically above C

The whole system is in a thin tube of oil, as shown in the diagram below.



The string connecting A and B has natural length 2 metres , and the tension in this string is $7e \text{ newtons}$ when the extension is $e \text{ metres}$.

The string connecting B and C has natural length 3 metres , and the tension in this string is $3e \text{ newtons}$ when the extension is $e \text{ metres}$.

18 (a) Find the extension of each string when the system is in equilibrium.

[3 marks]



- Use this model to answer part **(b)(i)** and part **(b)(ii)**.

- Show that during the subsequent motion the particle satisfies the differential equation

$$\frac{d^2x}{dt^2} + 9\frac{dx}{dt} + 20x = 0$$

[3 marks]

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[5 marks]

[illegible]

[illegible]

[1 mark]

END OF QUESTIONS



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ANSWER IN THE SPACES PROVIDED**



[illegible]

